## PLANAR PROPERTIES

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This article is about planar modular origami only and does not include the spirals. The properties are interesting and assist in model assembly.

- Every plane passes through the center of the model.
- Every plane intersects every other plane.
- If a planar is of degree $n$, i.e., there are $n$ planes, then the polygonal nature of the planes is of degree ( $n-1$ )
- Total number of units, $U=n(n-1)$.
- Angle at the base of the units, $\theta=360 /(n-1)$.
- Sum of number of sides of all facial polygons, $\Sigma S=4 U$.

The table below lists the properties of the various planars.

| Model Name | Number of Planes (n) | Nature of Planes (n-1) ${ }^{\circ}$ Polygon | Total \# of Units $\mathrm{U}=\mathrm{n}(\mathrm{n}-1)$ | Base Angle $\theta=360 /(n-1)^{\circ}$ | Underlying Polyhedron | Facial Polygons Or Rings | Sum Sides $\Sigma S=4 \mathrm{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| XYZ <br> (Cartesian Planes) | 3 | Digonal (2) | $3 \mathrm{x} 2=6$ | $360 / 2=180^{\circ}$ | Octahedron | $/ 8$ | 24 |
| WXYZ | 4 | Triangular (3) | $4 \times 3=12$ | $360 / 3=120^{\circ}$ | Cuboctahedron |  | $\begin{aligned} & 24+24 \\ & =18 \end{aligned}$ |
| VWXYZ | 5 | Quadrilateral (4) | $5 \times 4=20$ | $360 / 4=90^{\circ}$ | Pentagonal Gyrobicupola J31 |  | $\begin{aligned} & 30+40+10 \\ & =80 \end{aligned}$ |
| UVWXYZ | 6 | Pentagonal (5) | $6 \times 5=30$ | $360 / 5=72^{\circ}$ | Icosidodecahedron | $/ 20,12$ | $\begin{aligned} & 60+60 \\ & =120 \end{aligned}$ |
| TUVWXYZ | 7 | Hexagonal (6) | $7 \mathrm{x} 6=42$ | $360 / 6=60^{\circ}$ | No formal name* |  | $\begin{aligned} & 84+60+24 \\ & =168 \end{aligned}$ |
| STUVWXYZ | 8 | Heptagonal (7) | $8 \mathrm{x} 7=56$ | $360 / 7=51.4^{\circ}$ | No formal name* | $/ 28,14,14>2$ | $\begin{aligned} & 84+56+70 \\ & +14=224 \end{aligned}$ |
| RSTUVWXYZ | 9 | Octagonal (8) | $9 \times 8=72$ | $360 / 8=45^{\circ}$ | No formal name* | $/ 32,24,16>2$ | $\begin{aligned} & 96+96+80 \\ & +16=288 \end{aligned}$ |
| QRSTUVWXYZ | 10 | Nanogonal (9) | $10 \times 9=90$ | $360 / 9=40^{\circ}$ | Truncated Icosahedron-like |  | $\begin{aligned} & 180+60+120 \\ & =360 \end{aligned}$ |

Notes:

- It is best to make each plane in one colour to emphasize the planes and also for ease of assembly.
- As $n$ grows larger, $\theta$ should be made a degree or two smaller than the theoretical value above, to accommodate for the paper thickness.
* Please refer to assembly guide on next page.

The diagrams here are schematic and not to scale. The models are cut into two hemispheres, the black ring representing the equator.

A square dot with two arms ( ) represents a unit. Due to the abstract nature of the diagrams, a unit may not always appear as a straight line.

For the models TUVWXYZ and RSTUVWXYZ, the halves are not identical, so both halves have been shown. For the model STUVWXYZ, the two halves are identical. Part for the other half is shown in gray to illustrate its orientation with respect to the top half.


